

Alpha Sample Test 1 Solutions

Maximillian Roach

August 2024

Answer Key

1. 13

2. 2 and 9

3. 30

4. 14

5. $\frac{256}{3}\pi$

6. 6

Solutions

1. The legs of a right triangle are 5 and 12. What is the length of hypotenuse?

Solution: The Pythagorean Theorem is $a^2 + b^2 = c^2$, where a and b are the legs and c is the hypotenuse. Then $c^2 = 5^2 + 12^2 = 169$. Then $c = \sqrt{169} = 13$. Hence, the length of the hypotenuse is 13.

2. Two numbers sum to 11 and multiply to 18. What are the two numbers?

Solution 1: You can simply test different numbers that sum to 11 and find $2 + 9 = 11$ and $2 \cdot 9 = 18$, so 2 and 9 are our numbers.

Solution 2: Vieta's formulas state that if we have a quadratic equation of the form $ax^2 + bx + c = 0$, then the sum of the roots of the equation is equal to $-\frac{b}{a}$ and the product of the roots is equal to $\frac{c}{a}$. So we can create the quadratic equation $x^2 - 11x + 18 = 0$ since its roots will sum to 11 and have a product of 18 according to Vieta's formulas. Then we can factor $x^2 - 11x + 18 = (x - 9)(x - 2)$ to find that the roots are 2 and 9.

3. One-half some number is equal to that number minus 15. What is the number.

Solution: Let the number we're trying to find be x . By translating the problem into an equation, we have $\frac{1}{2}x = x - 15$. Multiplying both sides of the equation by 2 gives $x = 2x - 30$. Subtracting x from both sides gives $0 = x - 30$. Then we can add 30 to both sides, $x = 30$, our final answer.

4. A bathtub is emptied and begins being filled with water. Water comes into the bathtub at 5 liters per second, and leaves the bathtub at 3 liters per second. After 7 seconds, how much water is in the bathtub?

Solution: If 5 liters go into the bathtub per second and 3 liters leave every second, then the change in water every second is $5 - 3 = 2$ liters. Over 7 seconds, the change in volume of water in the tub is $2 \cdot 7 = 14$ liters. Because there is no water in the tub to begin with, the amount of water in the tub after 7 seconds is 14 liters.

5. What is the volume of a sphere with radius 4?

Solution: The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$. With $r = 4$, the volume is $V = \frac{4}{3}\pi 4^3 = \frac{4^4}{3}\pi = \frac{256}{3}\pi$.

6. If there are 4 people and each person shakes hands with every other person, then how many handshakes occur?

Solution 1: Let the 4 people be Person 1, Person 2, Person 3, and Person 4. Person 1 can shake hands with the other 3 people, producing 3 handshakes. After that Person 2 can shake hands with Person 3 and Person 4. We don't want to count their handshake with Person 1 again. This adds 2 more handshakes. Finally, Person 3 shakes hands with Person 4. Again, we don't want to count Person 3 shaking hands with Person 1 and Person 2 since we already counted those handshakes. This adds 1 more handshake. $3 + 2 + 1 = 6$ is the total number of handshakes.

Solution 2: Each of the 4 people shakes hands with each of the 3 other people for $3 \cdot 4 = 12$ handshakes. This overcounts handshakes. For instance, the handshake for Person 1 with Person 2 is counted and the handshake for Person 2 with Person 1 is counted. Each handshake is counted twice, so we can divide our original count by 2 to get the actual number of handshakes. $\frac{12}{2} = 6$