Alpha Sample Test 2 Solutions

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Answer Key

1. 2

2. 4

3. 3/7

4. 6

5. 6

6. 41

Solutions

1. Aarush buys 12 cookies at the grocery story. He gives 5 cookies to Aaron, 3 cookies to Adam, and 2 cookies to Aurora. How many cookies does he have left?

Solution: Giving 5 cookies away corresponds to subtracting 5. This means after giving cookies to Aaron, $12 - 5 = 7$ cookies are left. Giving 3 cookies away corresponds to subtracting 3. After giving cookies to Adam, $7 - 3 = 4$ cookies are left. Giving away 2 cookies corresponds to subtracting 2. After giving cookies to Aurora, $4 - 2 = 2$ cookies remain. Overall, this leaves 2 cookies.

2. What is 25% of 20% of 80?

Solution: 25% of 20% of 80 corresponds to $0.25(0.2(80))$. $0.2(80) = 16$. $0.25(16) = 4$.

3. There are 3 greens balls and 4 red balls in a bag. If a ball is randomly chosen from the bag, what is the probability that the ball is green?

Solution: The probability of an outcome is equal to the number of successful outcomes divided by the number of total outcomes. The number of total outcomes is 7 since there are 7 balls that could be chosen. There are 3 successful outcomes since there are 3 green balls in the bag. Therefore, the probability that a green ball chosen is 3/7.

4. How many arrangements of the digits 2007 are four digit numbers? For instance, 7200 is a valid arrangement.

Solution: The key mistake that one might make when calculating the number of arrangements is counting arrangements when 0 is the first digit. In the problem statement, it explicitly says that we want the arrangement to be a four-digit number. Having 0 as the first digit means the number is not a four-digit number. We can now count the number of successful arrangements with this in mind: 2007, 7002, 2700, 2070, 7020, 7020. This gives 6 successful arrangements. To ensure that there are exactly 6, we can consider the arrangements of N-N-Z-Z, where N represents a non-zero number (2 or 7)

and Z represents 0. Then the only valid arrangements of this (we can't have Z be first) are NZZN, NNZZ, NZNZ. For each of these there are two successful arrangements of 2007 since there are two ways to permute the nonzero numbers (2 and 7 are distinct).

5. Find the number of two-digit positive integers whose digits sum to 6?

Solution: Let's first consider the pairs of digits that sum to 6. These are 6-0, 5-1, 4-2, and 3-3. 6-0 only gives 60 since 06 isn't a two-digit positive integer. 5-1 corresponds to 51 and 15. 4-2 corresponds to 42 and 24. 3-3 only corresponds to 33. This gives 6 two-digit positive integers whose digits sum to 6.

6. A sequence begins with 2. To get the next element in the sequence, you add 3 to the previous element. So the sequence follows 2, 5, 8, ... What is the fourteenth element in the sequence?

Solution: The kth term can be written in form $3(k-1)+2$. This is because to get to the kth term you add 3 $k-1$ times. You don't add 3 any times for the first term and you add 3 for the first time for the second term. From this form, then 14th term is $3(14-1)+2 = 3(13)+2 = 39+2 = 41$.