Alpha Sample Test 3 Solutions

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Answer Key

1. 13.5

2. 86

3. 10π

4. 65

5. $9\pi + 54$

6. $6\pi + 30$

Solutions

1. It is snowing outside. 3 inches of snow accumulate every 2 hours. If it started snowing at 5:00 PM, how much snow is on the ground at 2:00 AM?

Solution: If 3 inches of snow accumulate every 2 hours, then the rate of snow falling is $\frac{3}{2}$ inches per hour. There are 9 hours between 5:00 PM and 2:00 AM. Therefore, the the number of inches of snow on the ground at 2:00 AM is $\frac{3}{2} \cdot 9 = \frac{27}{2} = 13.5$ inches.

2. Throughout the school year, Robbie has taken 4 tests. His scores on these tests were 70, 85, 86, and 73. Robbie will take one final test. If he wants his average score on the 5 tests to be 80, what score must he earn on the final test?

Solution: The average of a group of numbers is the sum of the numbers divided by the number of numbers.

$$Average = \frac{\text{sum of elements}}{\text{number of elements}}$$

The sum of Robbie's previous test scores is $70 + 85 + 86 + 73 = 314$. After Robbie takes the final test, he will have taken 5 tests, so the number of elements in the formula for average above is 5. If we let x be the score Robbie needs on the final test, than the sum of elements becomes $314 + x$ since its the sum of the scores on the previous tests plus the sum of the score on the final test. Then we have the equation

$$\frac{314 + x}{5} = 80$$

since the average score must be 80. We can multiply both sides by 5, $314 +$ $x = 400$. Subtracting 314 from both sides, $x = 86$, so the score Robbie must score on his final test is 86.

3. A circle has diameter 10. What is the circle's circumference?

Solution: The formula for the circumference of a circle is

$$C = 2\pi r$$

where r is the radius of the circle. By definition, the diameter is twice the radius, so the radius is half the diameter. So if the diameter is 10, the radius is $10/2 = 5$. Using the formula, the circumference is $C = 2\pi r = 2\pi(5) = 10\pi$.

4. Sherlock has a wood board with width 10 in. and length 15 in. If Sherlock wants the ratio of the width to the height to be 5:1, how many inches must he add to the width of the board?

Solution: Let x be the number of inches that must be added to the width. Then the new width is x + 10. If we want the ratio of the width to the height to be 5:1, then we have the equation

$$\frac{x+10}{15} = 5$$

We can multiply both sides by 15, $x + 10 = 75$. Subtracting 10 from both sides gives $x = 65$. Hence, the number of inches that must be added to the width of the board is 65 in.

5. A coffee table is composed of a rectangular portion with two half circles on the ends. If the half circles have radius 3 and the length of the rectangular portion is 9, what is the area of the surface of the coffee table?

Solution: The area of a circle is

$$A = \pi r^2$$

and the area of a rectangle is

$$A = lw$$

The area of a half circle is then one-half of the area of a $\frac{1}{2}(\pi r^2)$. The area of each half circle with radius 3 is then $\frac{1}{2}(\pi 3^2) = \frac{9}{2}\pi$. The combined area of the circles is $\frac{9}{2}\pi + \frac{9}{2}\pi = 9\pi$. Alternatively, we could have recognized that the half circles combine to a complete circle with radius 3, then use the formula. We are given that the length of the rectangle is 9. The width is the diameter of the half circles, $2 \cdot 3 = 6$. By the formula for the area of a rectangle, it has area $6 \cdot 9 = 54$. We can sum the areas we've found to get the total area, $9\pi + 54$.

6. What is the perimeter of the coffee table described in Problem 5?

Solution: The formula for the circumference of a circle is

$$C = 2\pi r$$

and the formula for the perimeter of rectangle is

$$2l + 2w$$

As mentioned in the previous solution, we can treat the two half-circles as one complete circle. Then the circumference of a circle with radius 3 is $2\pi(3) = 6\pi$. The rectangle has length 6 and width 9 so its perimeter is $2(6) + 2(9) = 30$. Therefore, the perimeter of the coffee table is $6\pi + 30$.