Delta Sample Test 1 Solutions

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Answer Key

1. 27

2. 18

3. $x = -2$ and $x = 0$

4. 4/5 hours or 48 minutes

5. 67

6. 14

Solutions

1. It's chilly outside and Matt has to decide what to wear. He has three hats: one striped, one polka dot, and one plain. The same is true of his scarfs, and his pairs of socks. How many combinations can he wear?

Solution: The choice of a hat, scarf, and pair of socks are all independent of each other. This means that we can multiple by the number of choices for each clothing item to get the total number of ways for Matt to choose an outfit. Because there are three choices for each clothing item, the answer is $3 \cdot 3 \cdot 3 = 27$.

2. But wait! Matt hates when his hat and scarf have the same pattern. So how many combinations can he wear now? (Use the information from problem 1)

Solution: The pair of socks still has 3 choices since it remains independent of the choices for the hat and scarf. We could directly count the number of combinations for which the hat and scarf don't have the same pattern, but a simpler method is to use complementary counting. We count the number of ways that our condition isn't satisfied, which is when the hat and scarf have the same pattern. We then subtract this from the total number of ways to choose a hat-scarf pair.
The number of unsuccessful pairs is 3, one for each of the three different patterns. Because there are 3 choices for the pattern for both the hat and the scarf, the number of ways to choose a hat-scarf pair without restriction is 9. Therefore, the number of ways to choose a hat-scarf pair such that they have different patterns is 6. Because the choice of the hat-scarf pair is independent of the choice of a pair of socks, the answer is the product of their choices, $6 \cdot 3 = 18$.

3. Find the values of x for which the equation $(x-1)^2+4(x-1)+3 = 0$ is true.

Solution 1: We can expand $(x-1)^2$ and $4(x-1)$, combine like terms and solve the resulting quadratic equation. $(x-1)^2 = x^2-2x+1$ and $4(x-1) = 4x-4$. Then $(x-1)^2+4(x-1)+3 = (x^2-2x+1)+(4x-4)+3 = x^2+2x$. We could use the quadratic formula to factor x^2+2x but its much simpler to factor it since we can just take out a factor of x. $x^2+2x = x(x+2)$. This reveals $x = 0$ and $x = -2$ are the solutions.

Solution 2: Notice that x-1 is present in both $(x-1)^2$ and $4(x-1)$, so we might be inclined to let $a = x - 1$. Then we have a much simpler quadratic $a^2 + 4a + 3$. This can be easily factored as $a^2 + 4a + 3 = (a+3)(a+1)$, which reveals $a = -3$ and $a = -1$. Using our substitution $a = x - 1$, we have $-3 = x - 1$ and $-1 = x - 1$. Adding 1 to both sides of both equations, we get $x = -2$ and $x = 0$.

4. Davin and Connor are washing their car. When Davin washes the car by himself, it takes him 2 hours. Connor is a little quicker, so when he washes the car by himself it only takes him 1 hour and 30 minutes. If Davin and Connor wash the car together, how long will it take them?

Solution: We can use the formula $distance = rate \cdot time$. In this case, the distance is 1 complete car washed and the rate is the number of cars washed for unit of time. When Davin washes the car by himself, it takes him two hours, which means his rate is 1/2 cars per hour. An easy way of realizing this is by letting $distance = 1$ (In this problem, we'll always let $distance = 1$ since Davin and Connor are never washing more than 1 car) and $time = 2$ since it takes Davin two hours. When Connor washes the car by himself, it takes him 1 hour and 30 minutes, which is 4/3 hours. By similar reasoning, his rate is 3/4 cars per hour. Then when Davin and Connor work together, the rate is the sum of their individual rates, $1/2 + 3/4 = 5/4$ cars per hour. To find the time it takes them when working together, we set up the $distance = rate \cdot time$, again letting $distance = 1$ since they're still washing only 1 car. But now the rate is 5/4 cars per hour. Then $1 = (4/5) \cdot time$, which reveals $time = 4/5$. Because the units of our equation is in hours, the answer is 4/5 hours, or 48 minutes.

5. Of the positive integers less than or equal to 100, how many are divisible by 2 or divisible by 3, but not divisible by both 2 and 3.

Solution: We can count the number of positive integers less than or equal to 100 that are multiples of 2, then the number that are multiples of 3. If we sum these, we correctly count the integers that are only multiples of 2 and that are only multiples of 3. But we overcount those that are multiples of 2 AND 3, since they are counted once in the first count and once in the sec-

ond count. Therefore, we must subtract these integers once so that they're counted once instead of twice.
The number of integers less than or equal to 100 that are divisible by 2 is $\lfloor \frac{100}{2} \rfloor = 50$, where $\lfloor x \rfloor$ denotes the greatest integer less than x, for all real x. This is the case because we're counting $2 \cdot 1, 2 \cdot 2, \cdots, 2 \cdot 50$, which is a list of length 50. This technique is better illustrated with 3 since 100 isn't evenly divisible by 3. The number of integers less than 100 that are divisible 3 is $\lfloor \frac{100}{3} \rfloor = 33$. $\frac{100}{3}$ is the maximum quotient that 3 can be multiplied such that the product is less than 100. This is because 100/3 is not an integer, so we're taking the greatest integer less than it to get the maximum quotient. With similar reasoning, the number of integers less than or equal to 100 that are divisible by 6 is $\lfloor \frac{100}{6} \rfloor = 16$. Then, the answer is $50+33-16 = 67$

6. Henri and Juliet want to meet to eat lunch. To minimize the time they have to travel, they chose a restaurant between their houses. They leave their houses at the same time and arrive at the restaurant at the same time. If Henri walks 3 miles per hour, Juliet walks 4 miles per hour, and it takes them 2 hours to reach the restaurant, what is the distance between their houses?

Solution: The distance between Henri's house and Juliet's house is the sum of the distance each person has to walk. We can use $distance = rate \cdot time$. Henri walks 3 miles per hour for 2 hours, so his distance is 6 miles. Juliet walks 4 miles per hour for 2 hours, so his distance is 8 miles. Then the answer is $8 + 6 = 14$.