Delta Sample Test 2 Solutions

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Answer Key

1. 353

2. 25/4

3. 12000

4. 6

5. 8/3

6. 32

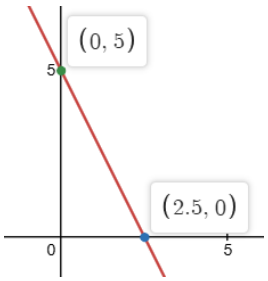
Solutions

1. A palindrome is a number that is the same when you reverse its digits. For example, 12321 is a palindrome. What is the smallest palindrome greater than 345?

Solution: You can obviously just keep going up by one and test to see if the number you're on is a palindrome, but there is a slightly more motivated approach. We can surmise that the least palindrome greater than 345 is in the 300s. Then the units digit must also be 3. We just have to decide what the tens digit. Well, we want to change the tens digit as little as possible, so we can let it be 5. There is also no other digit that the middle digit is dependent upon, so we know we can change it freely. This means the palindrome is 353. You can test for yourself that there exists no palindrome between 345 and 353.

2. Consider the line 4x + 2y = 10. It intersects with the y-axis at one point and with the x-axis at one point. What is the area of the triangle determined by these two intersection points and the origin.

Solution: We first want to find the intersection points with the y-axis and x-axis. Recall that the y-intercept (where something intersects with the y-axis) occurs when x = 0. Then we can let x = 0 to find the corresponding y-coordinate. $4(0) + 2y = 10$. Then $y = 5$. Therefore, the y-intercept is at (0, 5). Recall that the x-intercept occurs when y =0, so we can let y = 0 and find the corresponding x-coordinate. $4x + 2(0) = 10$, so $x = 5/2$. The triangle created is a right triangle since the x-axis and y-axis intersect at a right angle and the origin is a vertex of the triangle.



Then the the legs of the right triangle have length 5 and 5/2. Recall that

the area of a triangle is

$$A = \frac{1}{2}bh$$

so the area of this triangle is $\frac{1}{2}(5)(\frac{5}{2}) = \frac{25}{4}$.

3. Milton is making a map with a 1:300 ratio. This means an inch on the map corresponds to 300 inches in real life. If a region on Milton's map is 40 inches long, how long is the region in real life?

Solution: An inch on the map corresponds to 300 inches in real life, so 40 inches corresponds to $40 \cdot 300 = 12000$ inches.

4. A perfect number is a number who divisors (besides itself) sum to the number. For example, 28 is a perfect number. It's divisors that are not 28 are 1, 2, 4, 7, and 14. $1 + 2 + 4 + 7 + 14 = 28$, making 28 a perfect number. What is another perfect number?

Solution: 6 is a perfect number because its factors (that are not 6) are 1, 2, and 3. $1 + 2 + 3 = 6$.

5. Margaret's cup holds 8 fluid ounces of liquid. She adds 4 fl. oz. of coffee and 2 fl. oz. of milk in the cup and stirs. Then she drinks 1/3 of the liquid in the cup. How many fl. oz. of coffee are left in the cup?

Solution: After adding the coffee and milk, there is a total of 6 fl. oz. of liquid in the cup. Because there are 4 fl. oz. of coffee and 2 fl. oz. of milk, the ratio of coffee to milk is 2:1. This means that for every unit of liquid in the cup, $\frac{2}{3}$ is coffee and $\frac{1}{3}$ is milk. 1/3 of the liquid is taken out of the cup, which corresponds to 2 fl. oz. 2/3 of this is coffee. This means $2 \cdot \frac{2}{3} = \frac{4}{3}$ fl. oz. of coffee is removed from the cup. If the cup originally had 4 fl. oz. of coffee, then there remains $4 - \frac{4}{3} = \frac{8}{3}$ fl. oz. of coffee in the cup.

6. Elliot has 5 different donuts that he could eat for breakfast. He can eat any number of the donuts and leave the ones that he doesn't want. How many ways are there for Elliot to choose donuts to eat?

Solution: Observe that for each donut there two choices: (1) eat the donut, or (2) don't eat the donut. The choice of whether or not to eat each donut

is independent of the choices for the other donuts, so we can multiply the number of choices for each donut to get the total number of ways to choose which of the 5 donuts to eat. Because there are 2 choices for each donut, there are $2^5 = 32$ total ways to each the donuts.