Delta Sample Test 3 Solutions

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Answer Key

1. 385

2. 2

3. 8

4. 128/2187

5. $-\frac{4}{3}$

6. 9/5

Solutions

1. A grocery store worker is creating a pyramid of oranges to attract attention for the store's new sale. The top of the pyramid has 1 orange. The row after that has 4 oranges, and the row after that has 9 oranges. This continues until the pyramid has 10 rows. How many oranges are in the pyramid?

Solution: We are trying to sum the first 10 squares. Our sum is

$$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2$$

You could calculate this sum by hand, but that's really gross, so instead we can use the formula for the sum of the first n squares:

$$\frac{n(n+1)(2n+1)}{6}$$

Letting n = 10, we get

$$\frac{10(11)(21)}{6} = 385$$

2. How many times does the graph of $x^2 + y^2 = 4$ intersect the x-axis?

Solution: We can recognize that the equation represents the equation of a circle with radius 2 centered at the origin. The circle intersects with the x-axis twice, namely at (-2,0) and (2,0).

3. A smaller cube has side length 1/2 that of the larger cube. How much larger is the volume of the larger cube in comparison to the smaller cube?

Solution: The volume of a cube with side length s is

$$V = s^3$$

Let the larger cube have side length a. Then the volume of the larger cube is a^3 If the smaller cube has side length 1/2 that of the larger cube, then its side length is $\frac{1}{2}a$. Then the volume of the smaller cube is $(\frac{1}{2}a)^3 = \frac{1}{8}a^3$. To find how much larger (by a factor) the larger cube's volume is compared to the smaller cube's volume, we divide their volumes

$$\frac{a^3}{\frac{1}{8}a^3} = 8$$

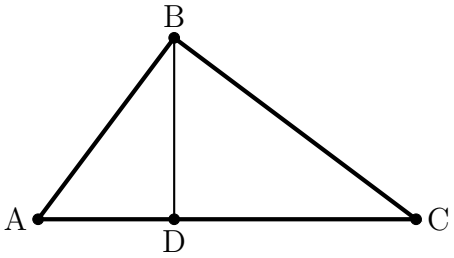
4. A certain basketball team has a 1/3 probability of winning any game they play. What is the probability that when they play 7 games, they lose every game?

Solution: If the probability of winning a game is 1/3, then the probability of losing a game is 2/3. The outcomes of the games are independent of one another, so we can multiply their probabilities (which is just multiplying 2/3 by itself seven times). $(2/3)^7 = 128/2187$

5. Give $4x - 5y = 13x + 7y$, what is the value of $\frac{x}{y}$?

Solution: We can first subtract 4x from both sides, $-5y = 9x + 7y$. Next we subtract 7y from both sides, $-12y = 9x$. We divide y by both sides to get and x/y term, $-12 = 9\frac{x}{y}$. Dividing both sides by 9 gives the answer, $\frac{x}{y} = \frac{-12}{9} = -\frac{4}{3}$

6. ABC is a right triangle with side lengths AB = 3, BC = 4, and AC = 5. A perpendicular bisector is drawn from B down to D, creating two more right triangles. What is the length of segment AD?



Solution: $\triangle ABD$ is similar to $\triangle ACB$. $\triangle ABD \sim \triangle ACB$ by AA similarity since they share $\angle A$ and both have right angles. When two triangles are similar, the ratio of corresponding sides is equal. This means

$$\frac{AD}{AB} = \frac{AB}{AC} = \frac{3}{5} \implies \frac{AD}{AB} = \frac{3}{5} \implies AD = \frac{3}{5}AB = \frac{3}{5}(3) = \frac{9}{5}$$