8-7-24 Omega Contest

Sample Test 3

Correct answers are worth 1 point. Incorrect and blank answers are worth 0 points. You are given 30 minutes to complete the test and no calculators are allowed.

1. ——————— How many positive integers less than 100 have a sum of digits divisible by 3?

2. ——————— What is the remainder when 2^{101} is divided by 3?

3. ——————— A donut shop has donuts with sprinkles, frosting, or both. 20 of the donuts have sprinkles, 30 have frosting, and 15 have both. How many donuts does the shop have?

4. ——————— Solve for x:

$$\log_4(\log_3(\log_2(x))) = 1$$

5. ——————— Alice is hosting a Super Bowl party. She doesn't know how many people will come, but that at least 1 person will attend, and at most 5 people. If she has 5 cans of soda, how many ways are there for her to distribute the cans so that everyone gets at least one can, for any number of people coming? For instance, if three people attend, she can distribute them 3-1-1. If four people attend, she can distribute them 2-2-1-0. Assume that the people are indistinguishable and she isn't including herself.

6. ——————— Circle O has radius 5. Two chords, AB and AC, are drawn parallel to each other. If EC = 3 and the distance between the two chords is 7, then what is the length of FB?